A mathematician is a machine for turning coffee into theorems. Erdős
Inductors

• What is an inductor? • An inductor always \( \frac{\partial}{\partial t} \) the changes in the
current through the circuit.

• Draw an RL circuit with a switch.
• Use Kirchhoff’s voltage law to get a differential equation.

\[ 0 > \frac{\partial}{\partial t} I_L \iff 0 > \frac{\partial}{\partial t} I_L \]

\[ 0 = R I - \frac{\partial}{\partial t} L - 3 \]

What is an inductor with a large self-inductance?
Derive the solution to the RL circuit.

Solution

\[ I = \left( \frac{\frac{3}{R} - 3}{R} - 1 \right) \frac{V}{R} \]

\[ \frac{3}{R} - 3 = \frac{l}{R} - e \]

\[ \left( \frac{\frac{3}{R} - 3}{R} \right) l = \frac{l}{R} - e \]

\[ \int_{0}^{\infty} \frac{R}{I} dt = \frac{l}{R} - 3 \]

\[ \int_{0}^{\infty} l dt = \frac{l}{R} - 3 \]

\[ \frac{\frac{3}{R} - 3}{R} \]

\[ (R) \left( \frac{\frac{3}{R} - 3}{R} \right) l = \frac{l}{R} - e \]

\[ \frac{\frac{3}{R} - 3}{R} \]

\[ \int_{0}^{\infty} l dt = \frac{l}{R} - 3 \]
Example

At \( t = \infty \), an inductor acts like an open switch. At \( t = 0 \), an inductor acts like a short circuit.

\[ \frac{\ell}{I} \int_0^T \frac{I_0}{I} \, dt = -\frac{\ell t}{I_0} \]

\[ \frac{\ell}{I} \int_0^T \Rightarrow R \Rightarrow I = -\frac{\ell t}{I_0} \]

For \( t > 0 \), \( I \) increases to \( \frac{\ell}{I_0} \).

Copy the circuit diagram, and describe changes in the circuit when the switches are opened/closed.

Close \( S_2 \) & \( S \) open is symmetrically.

Before

Close \( S_1 \): I decreases to \( \frac{\ell}{I_0} \) as shown.
Examples

1. Two circuits similar to the previous example have different inductors. Switch 1 is closed at $t = 0$ and then switch 2 is closed. Copy the graphs shown. Which circuit has the greater inductor?

2. A 12 V battery is in series with a 30 mH inductor and a 6 ohm resistor.

   a) How long does it take the current to reach $2(1-e^{-1})$ amps?
   
   b) Calculate the current at 2 ms.
   
   c) Sketch the voltages across the inductor and the resistor.
\[ (a) \quad I = 2 (1 - e^{-\frac{t}{3}}) = 0.66 A \]

\[ (b) \quad I = 2 \left( 1 - e^{-\frac{t}{3}} \right) \]

\[ I = \frac{6}{30} = \frac{2}{5} = \frac{2}{3} \]

\[ \frac{2}{3} \left( 1 - e^{-\frac{t}{3}} \right) \]

\[ 0.65 \text{ m/s} \]

\[ I = 2 \left( 1 - e^{-\frac{t}{3}} \right) \text{ and } \text{t is long} \]

\[ 2.5 \]

\[ 7 \]

\[ \frac{\mu}{4} = 2 \]

\[ 2.5 > 2 \]

\[ 1.5 \text{ Sam} \]