Applications of Gauss's Law

What lies behind us and what lies before us are tiny matters compared to what lies within us.
Emerson

Don't walk before me, I may not follow; Don't walk behind me, I may not lead; Walk beside me and be my friend.

$\Phi_E = E \cdot A$; $\Phi_E = \oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$

Symmetric Charge Distribution

- We want to select a surface such that one or more of the following conditions are met:
  1. The value of the E-field can be argued by symmetry to be constant over the surface.
  2. The dot product can be expressed as a simple algebraic product EdA because **E** and **dA** are parallel. $E \parallel dA \Rightarrow E \cdot dA = EdA$
  3. The dot product is zero b/c **E** and **dA** are perpendicular. $E \perp dA \Rightarrow E \cdot dA = 0$
  4. The field can be argued to be zero over the surface.

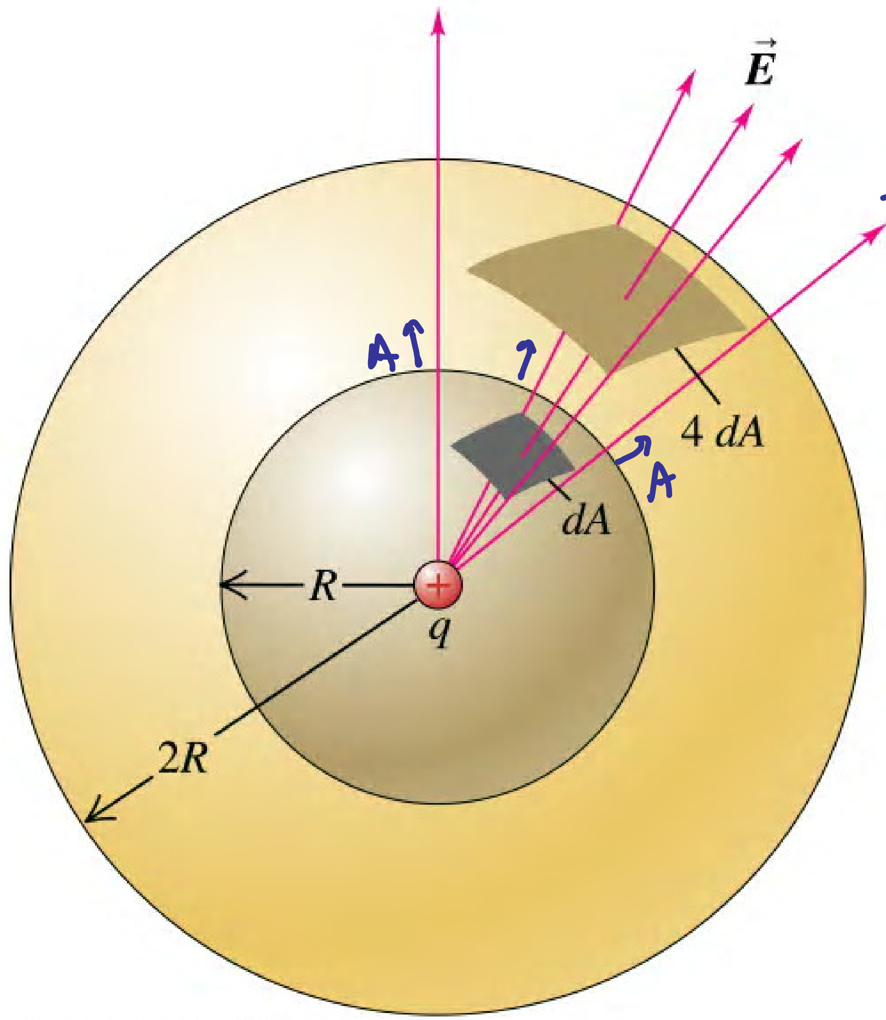
Examples, pages 750 - 754

1. Calculated the electric field due to a point charge, +q. $E \parallel dA$

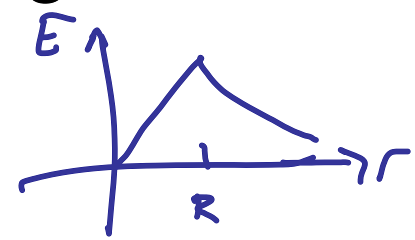
$$\oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

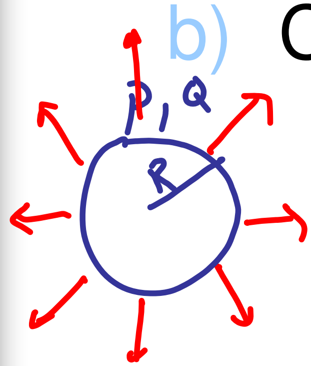
$$E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \qquad E = \frac{kq}{r^2}}$$



2. An insulating solid sphere of radius R has a uniform volume charge density ρ and carries a total positive charge Q.
   a) Calculate E-field outside sphere.
   b) Calculate E-field inside sphere.





a) $r > R$, $E = \frac{kQ}{r^2}$

b) $r < R$, $\oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$

$E(4\pi \cancel{r^2}) = \frac{Q r^{\cancel{3}}}{\epsilon_0 R^3}$

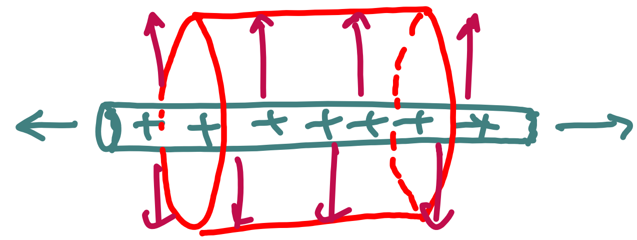
$\boxed{E = \frac{kQr}{R^3}}$

$\frac{q_{in}}{Q} = \frac{\cancel{\frac{4}{3}\pi} r^3}{\cancel{\frac{4}{3}\pi} R^3}$

$q_{in} = \frac{Q r^3}{R^3}$

3. A thin spherical shell of radius R has a total charge Q distributed uniformly over its surface. Find the E-field at
   a) points outside the shell $\boxed{E = \frac{kQ}{r^2}}$
   b) inside the shell $\boxed{E = 0}$, $q_{in} = 0$

4. Find the electric field a distance *r* from a line of positive charge of infinite length constant charge per unit length λ.



5. Find the E-field due to an insulating, infinite plane of positive charge with uniform surface charge density σ.

$$\oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

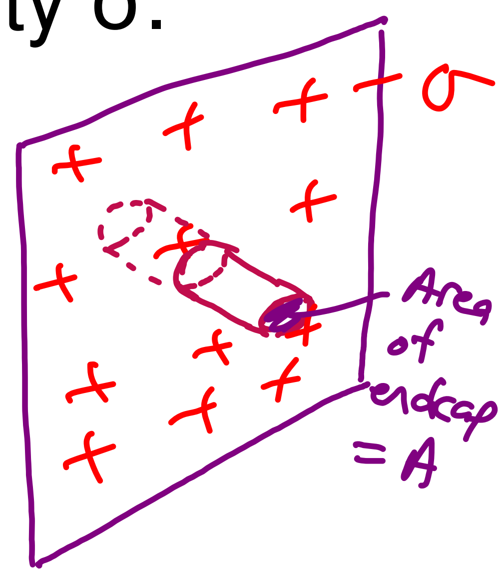
$$q_{in} = \sigma A$$

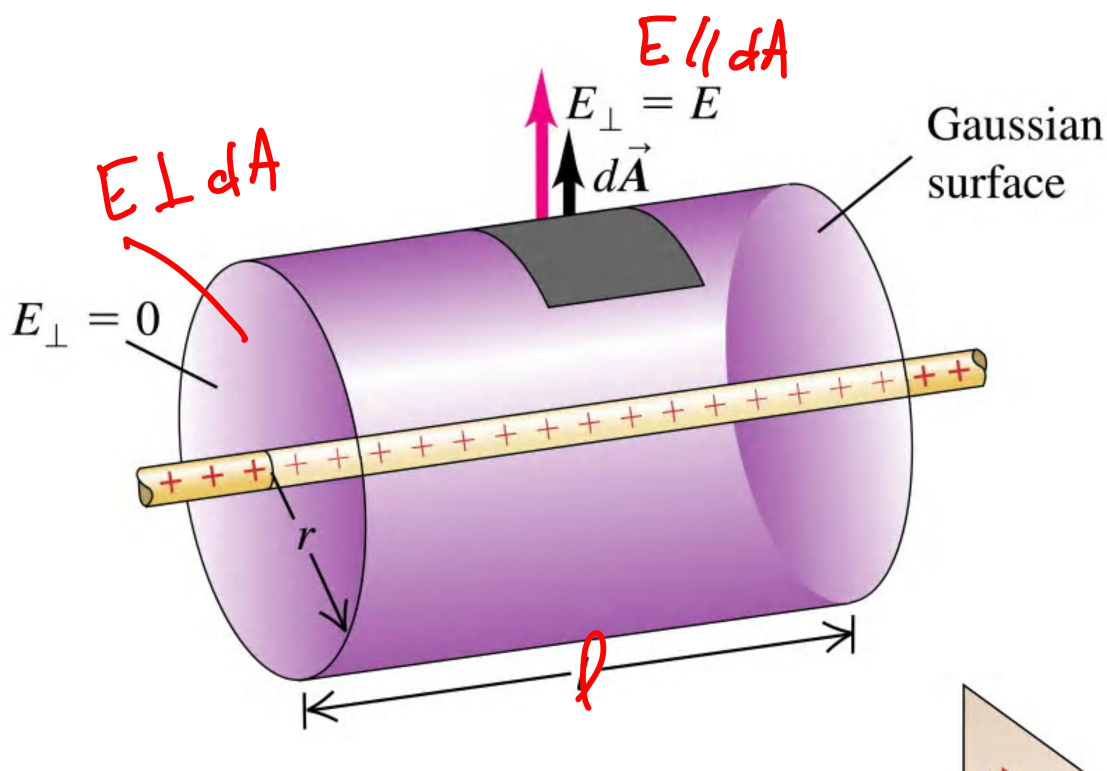
$$E(2\cancel{A}) = \frac{\sigma \cancel{A}}{\epsilon_0}$$

$$\sigma = \frac{q}{A}$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

sheet





4. $\oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$

$E(2\pi r \cancel{\ell}) = \frac{\lambda \cancel{\ell}}{\epsilon_0}$

$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k\lambda}{r}}$