AP Calculus BC
Lesson 5.5 Evaluating Integrals using substitution

1. Use the suggested *u*-substitution to evaluate each indefinite integral:

(a) $\int \frac{3x^2}{x^3+1}dx, \quad u = x^3 + 1$

(b) $\int 2x \cdot \sin(x^2)dx, \quad u = x^2$

(c) $\int \sin^3(x)\cos(x)dx, \quad u = \sin(x)$

(d) $\int \frac{2xdx}{\sqrt[3]{x^2+1}}, \quad u = x^2 + 1$

2. Evaluate $\int \cos(x)\sin(x)dx$ in three different ways:

(a) Make the substitution $u = \cos(x)$.

(b) Make the substitution $u = \sin(x)$.

(c) Use the identity $2\sin(x)\cos(x) = \sin(2x)$.

Are the answers equivalent? Why or why not?

3. Evaluate each definite integral:

(a) $\displaystyle\int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\,dx$

(b) $\displaystyle\int_0^{\sqrt{7}} x(x^2+1)^{1/3}\,dx$

4. Show that $\int \tan(x)dx = \ln\left|\sec(x)\right| + C$ by using a u-substitution on $\displaystyle\int \frac{\sin(x)dx}{\cos(x)}$.

5. Evaluate each integral in two different ways.

(a) $\int \sqrt{x}(1+x)dx$

(b) $\int x\sqrt{x+1}dx$

(c) $\int \frac{\cos(x)}{\sqrt{2+\sin(x)}}dx$

(d) $\int \frac{dx}{1+x}$

(e) $\int \frac{dx}{\sqrt{x}(1+x)}$

(f) $\int \tan^2(x)\sec^2(x)dx$

(g) $\int \frac{2^{\sqrt{x}}}{\sqrt{x}}dx$

(h) $\int \cot(x)dx$

(i) $\int \frac{dx}{x\ln(x)}$

6. Evaluate $\int \sec(x)dx$ by first multiplying by $\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}$ and then using an appropriate *u*-substitution.

Apply a similar method to evaluate $\int \csc(x)dx$.

7. Find each antiderivative in two different ways.

(a) $\int \csc^2(2\theta)\cot(2\theta)d\theta$

(b) $\int \sin^3(x)\cos^3(x)dx$

(c) $\int \sqrt{1+\sin^2(x)}\sin(x)\cos(x)dx$

(d) $\int \frac{\log(10x)dx}{x}$

(e) $\int \tan^3(x)\sec^4(x)dx$

(f) $\int \frac{2\cos(x)dx}{1+\sin^2(x)}$

(g) $\int \frac{dx}{\sqrt{9-x^2}}$

(h) $\int \frac{x^2}{\sqrt{x+3}}dx$