AP Calculus BC
Lesson 4.7 Optimization Problems

1. [1982AB6BC3] A tank with a rectangular base and rectangular sides is to be open at the top. It is to be constructed so that its width is 4 meters and its volume is 36 cubic meters. If building the tank costs $10 per square meter for the base and $5 per square meter for the sides, what is the cost of the least expensive tank?

2. A picture 7 ft high is placed on a wall with its base 9 ft above the eye level of an observer. How far from the wall should the observer stand to get the best view of the picture? (Consider that the view will be best when the angle subtended at the observer's eye is a maximum.)

3. Points A and B are opposite each other on shores of a straight river 3 km wide. Point C is on the same shore as B but 2 km down the river from B. The telephone company wishes to lay a cable from A to C. If the cost per kilometer of the cable is 25 percent more under the water than it is on land, what is the least expensive route for the cable?

4. Find the dimensions of the right circular cylinder of greatest volume that can be inscribed in a right circular cone with radius 5 cm and height 12 cm.

5. An automobile traveling at the rate of 30 ft./sec. Is approaching an intersection. When the automobile is 120 ft. from the intersection, a truck traveling at the rate of 40ft./sec. crosses the intersection. The automobile and the truck are on roads that are at right angles to each other. How long after the truck leaves the intersection are the two vehicles the closest?

6. Jane is 2 miles offshore in a boat and wishes to reach a coastal village 6 miles down a straight shoreline from the point on the shoreline nearest the boat. She can row at 2mph and can walk at 5mph. Where should she land her boat to reach the village in the least amount of time?

7. A cone is formed from a circle of paper by cutting out a sector with central angle θ and joining the exposed radii. What central angle maximizes the volume of the resulting cone?

8. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800m of wire at your disposal, what is the largest area you can enclose?

9. A right triangle whose hypotenuse is $\sqrt{3}$ meters long is revolved about one of its legs to generate a right circular cone. Find the radius, height, and volume of the cone of greatest volume that can be made in this way.

10. A plot of ground in the shape of a circular sector (think piece of pie) is to have a border of roses along the straight lines and tulips along the circle. If the area of the plot is to be 100 square meters, and roses cost $20 a meter and tulips $15 a meter, what is the least the flowers will cost?

11. What values of a and b cause $f(x) = x^3 + ax^2 + bx$ to have
    a) A local maximum at x = -1 and a local minimum at x = 3.
    b) a local minimum at x = 4 and a point of inflection at x = 1.

12. In constructing the new Trump Colosseum, projected to occupy the whole state of Rhode Island, the builder estimates the initial costs (buying Rhode Island, etc.) as 450 times the cost of the first floor. The second floor is projected to cost twice as much as the first floor, the third floor three times as much as the first floor, etc. What number of floors in the building will give the cheapest average cost per floor?